is overlooked by some of those who deprecate these foundational results, it does not follow that (1) and (2) have little or no philosophical interest.

Anyone who is interested in the foundations of mathematics, whether from mathematical or epistemological motives, must come to understand how the connection (exploited by Descartes) between algebra and geometry is possible. How does it happen that one can do geometry via algebra, that one can co-ordinatise a geometric plane, and does it always happen that one can do so? Here Desargues's theorem provides the requisite condition for obtaining a 'nice' (i.e. associative) coordinate set and (1) demonstrates that even then, the coordinates are not nice enough. For only Pappus's theorem (or some equivalent) guarantees that the multiplication of coordinates is commutative. Thus (1) and associated results provide a startling and profound insight into fundamental aspects of geometry. Since philosophical interest in geometry has roots which go back at least as far as the interest in logical systems, it is difficult to see why Goodstein peremptorily dismisses the former in favour of the later.

The Skolem paradox is significant, as mentioned by Goodstein, because it has generated a whole area of research into nonstandard models and some concomitant philosophical discussion of the relation between a formal system and the subject matter and objects it deals with. But (2), the undecidability of Desargues's theorem, has similarly generated one of the most active fields of research in geometry: the discovery and classification of non-Desarguesian planes. This activity too has been accompanied by a good deal of discussion of the 'ontological' status of geometric objects and it has illuminated the older discussions of Euclidean vs. non-Euclidean geometry. Again, Goodstein's neglect seems unwarranted.

None of the foregoing is intended to disparage what Goodstein does say about the logical 'incompleteness theorems' it is only intended to correct a misleading impression and to indicate, for it is not appropriate in such a note to amplify, what Goodstein could have said about the more classical results.

ARTHUR I. FINE

REPLY TO MR FINE'S NOTE

IT is true that I overlooked a misprint in the third line of the second paragraph of my paper where the first word should be 'without', not 'and'.

It was not my intention to undervalue the importance of the proof of independence of Desargues's theorem and I accept the criticism that I could have said more about the concept of independence.

R. L. GOODSTEIN

REPLY TO G. A. BARNARD

MY article[1] does not 'hinge' on the statement singled out by Professor Barnard. This is admitted by him, apparently unconsciously, when he goes on to serious discussion of my distinction between each and all. The statement in question is a provocative aside, and I am glad Professor Barnard has responded.

If we put any notion of non-enumerability out of our minds, Skolem's theorem (also König's proof of 1905) shows that the real numbers are enumerable. Barnard's position is similar to that of veterinarian B who objected to A's findings on the horse because A had not considered the unicorn. My article pointed out a defect in the optical instrument used in unicorn sightings. To translate, all proofs of the existence of the transfinite rely on a simple contradiction in terms, namely, there is an end to an infinity. Since any approach to the transfinite must get beyond the enumerably infinite, it is at least as clear to me as Hilbert and Bernays's *Grundlagen* is to Barnard that there is no transfinite. After all, even metamathematicians are now forced to admit that their proofs and beliefs are ultimately intuitive. And how long did chemistry believe, with considerable success, in the existence of the undissociated molecule in electrolytic solutions before realising there is no such animal? It is not that, in the delightful phrase of Dr Lakatos, I am barring Barnard's unicorn because it is a monster—I am barring it because it does not exist within mathematics.

R. J. Diamond

[1] 'Each and All,' this *Journal*, 1964, 14, 351.