compounded with the normal 'forward information' to yield a precognitive insight on which the percipient sometimes *acts*? O. M. P. Bilaniuk, V. K. Deshpande and E. C. G. Sudarshan [1] have argued that a 'reversal of time' for a particle or a Lorentz transformation signifying a change of the sign of energy may conveniently be interpreted as a change in the description of the process. By an appropriate choice of the frame of reference, we say, for instance, that the particle is 'absorbed' instead of 'emitted'. The argument seems to be confined to Lorentz invariance and particles. It sheds no conceivable light on precognition occurring in an ordinary terrestrial setting in which neither the 'direction of time' nor memory can be regarded as a trivial choice by a *single* observer of a 'frame of reference'. R. W. Fuller and J. A. Wheeler [2] have shown that, with a relativistic description, even a multiply-connected topology of space-time may not provide for a particle travelling with the speed of light outpacing another particle of equal speed by taking a shorter path through a 'wormhole'. Some enthusiasts [3] have suggested that space-time description is fatally macroscopic and must be renounced in the microphysical world. It is impossible to extract a detailed hypothesis about ESP from so vague and sweeping a suggestion. I maintain that no physical theory of ESP proposed so far has successfully or even plausibly accounted for *all* the principal known facts about telepathy, clairvoyance, and precognition.

C. T. K. Chari

Note on Goodstein's 'The Significance of Incompleteness Theorems'

In 'The Significance of Incompleteness Theorems' (this *Journal*, 1963, **14**, pp. 208-220) R. L. Goodstein contrasts the classical 'incompleteness theorems' of geometry (and, he should have added, algebra) with the foundational results which stem mainly from the work of Gödel and Löwenheim. He elucidates this latter work expertly and indicates both the source and the error of several common misinterpretations. In doing so, however, he pays scant attention to the classical results and not only states them incorrectly but makes a hasty evaluation of their significance. Goodstein asserts (p. 208) that an axiom system for projective planes which consists of the incidence axioms plus Pappus's theorem is incomplete, because Desargues's theorem is undecidable in the system. But it is the remarkable (and well known) result of Hessenberg (*Math. Annalen* 1905, **61**) that in the presence of the axioms of incidence, Pappus's theorem implies Desargues's theorem. What Goodstein may have meant is either that (1) Pappus's theorem is undecidable on the basis of the incidence axioms and Desargues's theorem, or that (2) Desargues's theorem is undecidable given only the incidence axioms. This brings us to Goodstein's evaluation, which is simply to deny the philosophical importance of these geometrical 'incompleteness theorems'. While it is true, as Goodstein points out, that the incompleteness discovered by Gödel *et al* differs from (1) and (2) in being essentially unremovable (for sufficiently rich, recursively axiomatisable systems) and while, as Goodstein correctly remarks, this fact

[1] *Amer. J. Physics*, 1962, **30**, 718-23

[2] *The Physical Review* (2), 1962, **128**, 919-29; Cf. J. A. Wheeler, *Reviews of Modern Physics*, 1962, **34**, 873-92

[3] See, for instance, E. J. Zimmerman, *Amer. J. Physics*, 1962, **30**, 97-105

is overlooked by some of those who deprecate these foundational results, it does not follow that (1) and (2) have little or no philosophical interest.

Anyone who is interested in the foundations of mathematics, whether from mathematical or epistemological motives, must come to understand how the connection (exploited by Descartes) between algebra and geometry is possible. How does it happen that one can do geometry via algebra, that one can co-ordinatise a geometric plane, and does it always happen that one can do so? Here Desargues's theorem provides the requisite condition for obtaining a 'nice' (i.e. associative) coordinate set and (1) demonstrates that even then, the coordinates are not nice enough. For only Pappus's theorem (or some equivalent) guarantees that the multiplication of coordinates is commutative. Thus (1) and associated results provide a startling and profound insight into fundamental aspects of geometry. Since philosophical interest in geometry has roots which go back at least as far as the interest in logical systems, it is difficult to see why Goodstein peremptorily dismisses the former in favour of the later.

The Skolem paradox is significant, as mentioned by Goodstein, because it has generated a whole area of research into nonstandard models and some concomitant philosophical discussion of the relation between a formal system and the subject matter and objects it deals with. But (2), the undecidability of Desargues's theorem, has similarly generated one of the most active fields of research in geometry : the discovery and classification of non-Desarguesian planes. This activity too has been accompanied by a good deal of discussion of the 'ontological' status of geometric objects and it has illuminated the older discussions of Euclidean vs. non-Euclidean geometry. Again, Goodstein's neglect seems unwarranted.

None of the foregoing is intended to disparage what Goodstein does say about the logical 'incompleteness theorems' it is only intended to correct a misleading impression and to indicate, for it is not appropriate in such a note to amplify, what Goodstein could have said about the more classical results.

ARTHUR I. FINE

REPLY TO MR FINE'S NOTE

IT is true that I overlooked a misprint in the third line of the second paragraph of my paper where the first word should be 'without', not 'and'.

It was not my intention to undervalue the importance of the proof of independence of Desargues's theorem and I accept the criticism that I could have said more about the concept of independence.

R. L. GOODSTEIN

REPLY TO G. A. BARNARD

MY article[1] does not 'hinge' on the statement singled out by Professor Barnard. This is admitted by him, apparently unconsciously, when he goes on to serious discussion of my distinction between each and all. The statement in question is a provocative aside, and I am glad Professor Barnard has responded.