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What you always wanted to know about Bohmian mechanics but were afraid to ask

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Abstract: Bohmian mechanics is an alternative interpretation of quantum mechanics. We outline the main characteristics of its non-relativistic formulation. Most notably it does provide a simple solution to the infamous measurement problem of quantum mechanics. Presumably the most common objection against Bohmian mechanics is based on its non-locality and its apparent conflict with relativity and quantum field theory. However, several models for a quantum field theoretical generalization do exist. We give a non-technical account of some of these models.

Kexwords: Bohmian mechanics, de Broglie-Bohm theory, Interpretation of quantum mechanics, quantum field theory, theory generalization

1 Introduction

This note reviews Bohmian mechanics, an alternative interpretation (or modification) of quantum mechanics. Bohmian mechanics reproduces all predictions of quantum mechanics but introduces a radically different perception of the underlying processes. Like most alternative interpretations it is not distinguishable from standard quantum mechanics by e.g. any experimentum crucis.

We start out by a few historical remarks in Sec. 2 before we outline the main characteristics of its non-relativistic formulation in Sec. 3. Here we put special emphasis on the status of "observables" other than position. However, the most important feature of the theory is its solution to the infamous measurement problem of quantum mechanics (see Sec. 3.3).

We then turn to the question of relativistic and quantum field theoretical generalizations of the theory. Several such generalizations do exist and in Sec. 4 we give a non-technical account of some of these models. We also address the question

what it actually means to "generalize" a theory and make a little digression to the field of "intertheory relations".
However, before we get started, we would like to make some general remarks concerning the interpretation of quantum mechanics. These may help to put the debate on Bohmian mechanics into the wider context.

1.1 Reflections on the interpretation of quantum mechanics

The interpretation of quantum mechanics has been discussed ad nauseam and the engagement with it can be a frustrating and disappointing business. This subject matter continues to produce an endless stream of publications[1] and nobody can reasonably expect this issue to be settled in the future. So much the worse, the different camps stand in fierce opposition and one gets the impression that this is an other obstacle for reaching substantial progress.
However, what do we actually mean by "progress"? Perhaps, in a situation like this, we need to reconsider our criteria and standards for progress and success. Given that the foundation of quantum mechanics has a smooth transition to philosophy we may learn something from a similar debate there.
Chapter 15 of Bertrand Russell's little book *The Problems of Philosophy* (1912) is titled *The Value of Philosophy* and starts with a remark which applies just as well to the interpretation of quantum mechanics:

> "[W]hat is the value of philosophy and why it ought to be studied. It is the more necessary to consider this question, in view of the fact that many men, under the influence of science or of practical affairs, are inclined to doubt whether philosophy is anything better than innocent but useless trifling, hair-splitting distinctions, and controversies on matters concerning which knowledge is impossible."

And indeed, many practically minded physicists regard the interpretation of quantum mechanics as pointless since no direct applications follow from it.
Russell continues, that although philosophy does aim at "knowledge which gives unity and system to the body of the sciences", it admittedly had little success in this respect and could only answer very few of its questions definitely. However, more important than the answers are the questions it asks:

> "Philosophy is to be studied, not for the sake of any definite answers to its questions since no definite answers can, as a rule, be known to be true, but rather for the sake of the questions themselves; because these questions enlarge our conception of what is possible, enrich our intellectual imagination and diminish the dogmatic assurance which closes the mind against speculation (...)"

Now, rated by this measure, the debate on the interpretation of quantum mechanics is a story of spectacular success indeed. Agreed, only few questions have been

[1] Cabello (2000) gives a bibliographic guide to the foundation of quantum mechanics (and quantum information) and collects more than 10^5 entries.

settled ultimately, but every alternative interpretation enlarges "our conception of what is possible".[2] And this is exactly what Bohmian mechanics does as well. It enriches our conception of what the quantum world may be.

2 Some history

Bohmian mechanics was first developed by Louis de Broglie! Therefore we will use the name "de Broglie-Bohm theory" in the remainder of this paper. Some basic concepts of the theory were already anticipated in de Broglie's dissertation in 1924 and his talk on the 5th Solvay meeting in October 1927 contained an almost complete exposition of the theory – called the "pilot wave theory" (théorie de l'onde pilote) by him (Bacciagaluppi and Valentini 2006). For reasons which are not entirely clarified yet the theory fell into oblivion until David Bohm developed it independently in 1951 (Bohm 1952). However, the reception of this work was unfriendly, to say the least. See e.g. Myrvold (2003) for the early objections against the de Broglie-Bohm theory.

Since the 70s John Bell was one of the very few prominent physicists who stood up for the theory. Many papers in his anthology (Bell, 2004) use the de Broglie-Bohm theory and the stochastic collapse model by Ghirardi, Rimini and Weber (1986) as an illustration of how to overcome the conceptual problems of quantum theory. The de Broglie-Bohm theory is even closely related to Bell's most important discovery, the Bell inequality. It was the non-locality of the de Broglie-Bohm theory which inspired him to develop this result.

Interestingly, during the 60s and most of the 70s even Bohm himself had only little interest in his theory. Only since the late 70s he and his group (B. Hiley, Ch. Dewdney, P. Holland, A. Kyprianidis, Ch. Philippidis and others) at Birkbeck College in London started to work on that field again. They referred to the theory as "ontological" or "causal" interpretation of quantum mechanics. Since the 1990th some new groups and researchers joined the field (D. Dürr, S. Goldstein and N. Zanghi, A. Valentini, G. Grübl and others) and it came to the formation of different schools. Dürr, Goldstein and Zanghi (1992) coined the term "Bohmian mechanics" which stands for a specific reading of the theory. While mathematically equivalent to Bohm's exposition in 1952, it is influenced by Bell's (and also de Broglie's) presentation of the theory (e.g. it puts no emphasis on the "quantum potential"[3]).

[2] The above-mentioned should not be misconceived as a license for arbitrary speculations. The possible answers still have to come under scrutiny.

[3] It should be noted that while all of the before mentioned Bohm students use the quantum potential formulation, the presentation of the theory in Bohm and Hiley (1993) and Holland (1993) shows differences nevertheless. In addition changed also Bohm's own interpretation of the theory in the course of time. However, this is clearly not unusual and by no means specific to the de Broglie-Bohm theory. We just mention this point here to call into attention that – given these different readings of the theory – talking about *the* "de Broglie-Bohm theory" may need further qualification.

Researchers who want to stay away from this debate (or who entertain their own sub-variant) are usually identified by calling the theory "de Broglie-Bohm theory", "de Broglie-Bohm pilot wave model" or any similar permutation of the key words.

3 The non-relativistic formulation

The key idea of the (non-relativistic) de Broglie-Bohm theory (de Broglie 1927, Bohm 1952) is to describe a physical system not by the wavefunction, ψ, alone but by the couple of wavefunction *and* configuration, i.e. the position, Q_i, of the corresponding objects (e.g. electrons, atoms, or even macroscopic entities).

$$\boxed{\begin{array}{rcl} \psi & \rightarrow & (\psi, Q_i) \\ \text{quantum mechanics} & \rightarrow & \text{de Broglie-Bohm theory} \end{array}}$$

The theory is now defined by three postulates which will be explained in the following[4]:

1. The wavefunction satisfies the usual Schrödinger equation

$$i\hbar\frac{\partial\psi}{\partial t} = H\psi$$

2. The particle velocities (a real vector field on configuration space) are given by the so-called *guidance equation*:

$$\frac{dQ_k}{dt} = \frac{\nabla_k S(Q(t))}{m_k} \tag{1}$$

   With $Q(t) = (Q_1(t), \cdots, Q_N(t))$ the configuration of the system, m_k denotes the mass of particle k, ∇_k is the nabla operator applied to its coordinates and S the phase of the wavefunction in the polar representation $\psi = Re^{\frac{i}{\hbar}S}$.

3. The position-distribution, ρ, of an ensemble of systems which are described by the wavefunction, ψ, is given by $\rho = |\psi|^2$. This postulate is called the *quantum equilibrium hypothesis*.

Postulate 1 shows that ordinary quantum mechanics is embedded in the de Broglie-Bohm theory and that everything which is known about solutions of the Schrödinger equation remains valid and important. The de Broglie-Bohm theory is sometimes called a "hidden variable" theory since it supplements quantum mechanics with additional variables, i.e. the particle positions. However, this terminology is a bit awkward since the positions are not really "hidden".

[4] More detailed expositions of the de Broglie-Bohm theory can be found in Holland (1993), Bohm and Hiley (1993), Cushing (1994), Dürr (2001) Goldstein (2002) and Passon (2004).

Postulate 2 equips the particles with a dynamic which depends on the wavefunction. Metaphorically speaking the quantum particles are "riding" on (or guided by) the ψ-field.Thus the particles are moving on continuous trajectories and possess a well defined position at every instant. The proof for global existence of the Bohmian trajectories is given by Berndl *at al.* (1995) and was later extended by Teufel and Tumulka (2005).
The form of the guidance equation can be easily motivated.[5] One may take the classical relation between velocity (v), current (j) and density (ρ):

$$v = \frac{j}{\rho} \tag{2}$$

and inserts the quantum mechanical probability current, j, and the probability density ρ:

$$\begin{aligned} j &= \frac{\hbar}{2m_k i}\left[\psi^*(\nabla_k \psi) - (\nabla_k \psi^*)\psi\right] \\ \rho &= |\psi|^2 \quad . \end{aligned}$$

A different motivation of the guidance equation – based on symmetry arguments – is given in Dürr *et al.* (1992).
The above equation applies only to spinless particles. However, the generalization to fermions (or arbitrary spin) is straightforward. One only needs to consider solutions of the Pauli equation $(\psi_1, \psi_2)^t$ and arrives at the guidance equation 2 with the modified current:

$$j = \sum_a \left(\frac{\hbar}{2mi}(\psi_a^* \nabla \psi_a - \psi_a \nabla \psi_a^*) - \frac{e}{mc} A \psi_a^* \psi_a \right)$$

Postulate 3 is needed for the de Broglie-Bohm theory to reproduce all predictions of quantum mechanics. The continuity equation of quantum mechanics $(\frac{\partial |\psi|^2}{\partial t} + \nabla\left(|\psi|^2 \cdot \frac{\nabla S}{m}\right) = 0)$ ensures that any system will stay $|\psi|^2$ distributed if the quantum equilibrium hypothesis holds initially. The quantum equilibrium hypothesis provides the initial conditions for the guidance equation which make the de Broglie-Bohm theory to obey Born's rule in terms of position distributions. Since all measurements can be expressed in terms of position (e.g. pointer positions) this amounts to full accordance with all predictions of ordinary quantum mechanics.
Further more ensures the quantum equilibrium hypothesis that the de Broglie-Bohm theory does not allow for an experimental violation of Heisenberg's uncertainty principle notwithstanding the well defined position the particles possess in principle (Valentini 1991).
However, while it is ensured that the quantum equilibrium hypothesis is satisfied for a configuration which is $|\psi|^2$ distributed once, it is by no means clear why any configuration should be accordingly distributed initially. At first this seems like

[5] However, its form is not unique. One can add an arbitrary divergence-free vector-field and arrive at the same statistical predictions (Deotto and Ghirardi 1998).

a very specific requirement which needs e.g. very special initial condition of the universe. If the problem is viewed this way, it would be more appealing to have a dynamical mechanism which explains why $\rho \neq |\psi|^2$ distributed systems *evolve* into a quantum-equilibrium distributed configuration. This approach is explored in Valentini (1991, 1992) who claims that the dynamics of the de Broglie-Bohm theory gives rise to a relaxation into an approximate (i.e. coarse grained) equilibrium distribution for an enlarged set of initial configurations. However, there exists a more convincing approach to justify the quantum equilibrium hypothesis. Work by Dürr *et al.* (1992) shows, that the quantum equilibrium hypothesis follows by the law of large numbers from the assumption that the initial configuration of the universe is "typical" for the $|\Psi|^2$ distribution (with Ψ being the wavefunction of the universe). This derivation resembles the way Maxwell's velocity distribution for a classical gas follows from the "typicality" of the phase-space configuration of the corresponding gas (Dürr *et al.* 2004). According to this view the quantum equilibrium hypothesis is no postulate of the de Broglie-Bohm theory but can be derived from it.[6]

3.1 A remark on the quantum potential

While the above presentation introduced the guidance equation as fundamental, the original work of Bohm (1952) (and later also e.g. Holland 1993) introduced the notion of a "quantum potential". For the phase of the wavefunction the following equation holds:

$$-\frac{\partial S}{\partial t} = \frac{(\nabla S)^2}{2m} + V - \frac{\hbar^2 \nabla^2 R}{2mR} \quad . \tag{3}$$

Due to the similarity with the classical Hamilton-Jacobi equation (for the action S) the term $\propto \hbar^2$ has been baptized "quantum potential". Within the Hamilton-Jacobi theory the particle velocity is constraint to $m \cdot v = \nabla S$, which corresponds to the guidance equation of the de Broglie-Bohm theory. If one adopts the quantum potential formulation the motion along the Bohmian trajectories can be thought of as taking place under the action of a novel "quantum-force".
However, the guidance equation can be motivated e.g. by symmetry arguments (Dürr *et al.* 1992) and needs no recourse to the Hamilton-Jacobi theory. In Goldstein (1996) it is argued that moreover the quantum potential formulation is misleading since it suggests that the de Broglie-Bohm theory is just classical mechanics with an additional potential (or force) term. But the de Broglie-Bohm theory is a first-order theory (i.e. the velocity is constrained by the position already) and this important trait is disguised in the quantum potential formulation. Whether this ambiguity in the formulation of the de Broglie-Bohm theory should be viewed as a substantial debate or a secondary matter depends on the context. These two readings of the theory have certainly a great deal in common and

[6] At the risk of being imprecise we gave only a short sketch of the different strategies to motivate the quantum equilibrium hypothesis. For details the reader is referred to the original literature.

in discussing the de Broglie-Bohm approach in contrast to standard quantum mechanics the distinction between these different schools is usually irrelevant. However, more detailed discussions which involve subtleties regarding e.g. the status of the wavefunction, particle properties and philosophical implications of the de Broglie-Bohm theory in general have to pay attention to these differences.

3.2 Characteristic features

After the definition of the theory we want to discuss some of its characteristic features and try to put them into the wider context.

Determinism

The de Broglie-Bohm theory is deterministic since the wavefunction and the configuration at a given time fix the time evolution of the system uniquely. However, given the quantum equilibrium hypothesis the predictive power of the theory is not enlarged compared to ordinary quantum mechanics. All predictions of the theory remain probabilistic but in contrast to ordinary quantum mechanics, the randomness is arising from averaging over ignorance.
However, it should be noted that to many adherents of the de Broglie-Bohm theory, determinism is not the key feature of the theory. For example Bohm and Vigier (1954) have developed a hidden variable model which contains a stochastic background-field and in a later section we will discuss a field-theoretical generalization of the de Broglie-Bohm theory which also contains stochastic effects. Moreover do many "Bohmians" appreciate the GRW model which includes a stochastic term into the Schrödinger equation to describe the wavefunction collapse. Short but to the point: not the indeterminism of quantum mechanics but rather its vague account of the measurement process created discomfort with the ordinary formulation and inspired the development of these alternative models.

"Complementarity" dispensable

Many quantum phenomena (e.g. interference effects) need both, the wave and particle aspect of matter for their explanation. The notion of "complementarity" was developed as an attempt to justify this common use of mutually contradictory concepts. Within the de Broglie-Bohm theory matter is described by a wave-like quantity (the wavefunction) *and* a particle-like quantity (the position). Hence, the notion of complementarity is not needed.

Non-locality

Since the wavefunction is defined on the configuration space, the guidance equation of a N-particle system links the motion of every particle to the positions of the other particles at the same time. In principle the particles can influence each other over arbitrary distances. However, this non-locality is needed in order to

explain the violation of Bell's inequality. Moreover ensures the quantum equilibrium hypothesis that the correlation of space-like separated particles can not be used for faster than light communication (Valentini 1991). Finally does the non-locality of the de Broglie-Bohm theory vanishes if the state is not entangled. Whether this non-locality is viewed as an unacceptable feature depends on the attitude towards the problem of non-locality in quantum mechanics in general. Following the work of Bell and the experimental confirmation of quantum mechanics in tests of the Bell inequality it became widely accepted that quantum mechanics itself is "non-local". However, the precise meaning of the term "non-local" is far from being unique and their exists a vast literature on that topic. A thorough discussion of that issue is far beyond the scope of the present paper (see e.g. Cushing 1987). However, one can reasonably state, that the "non-locality" of the de Broglie-Bohm theory is more explicit (i.e. dynamical) than the "non-separability" of ordinary quantum mechanics.
Be that as it may, given that the de Broglie-Bohm theory is a reformulation of non-relativistic quantum mechanics, any action-at-a-distance should be no threat anyway. It is turned into an objection against the theory if one argues that no "Bohm-like" relativistic or quantum field theoretical generalization of the theory can be given. In Sec. 4 we will discuss the existing models for such generalizations.

"Measurements" deserve no special role

The main merit of the de Broglie-Bohm theory is its solution to the measurement problem. This theory treats "measurements" like any other interactions or experiments. This allows a reply to the frequent complaint that the trajectories of the de Broglie-Bohm theory violate the rule "Entia non sunt multiplicanda praeter necessitatem" which is usually attributed to William of Ockham ("Ockham's razor"). While the trajectories are additional entities indeed, any "measurement postulate" or the like becomes unnecessary. Given the importance of this point we devote Section 3.3 to a more detailed discussion of the measurement-problem and how it is solved by the de Broglie-Bohm theory.

"Observables" other than position and contextuality

Much more important than being deterministic or having particle trajectories is the novelty of the de Broglie-Bohm theory with regard to the status of "observables" other than position. Within ordinary quantum mechanics the identification of "observables" with linear Hilbert space operators is usually regarded as the key innovation. Their non-commutativity is believed to be the mathematical embodiment of the deep epistemological lesson quantum mechanics teaches us.
The de Broglie-Bohm theory takes a different route. First, it includes the particle positions (which are described by real coordinates, and not by some operator) into the state description. Second, it distinguishes these variables, i.e. the outcome of every experiment is determined by the wavefunction and the configuration. Note, that this holds also for experiments which are supposed to "measure" quantities like energy, angular momentum, spin etc. There are no "hidden variables"

or continuous functions which correspond to the "actual" values of these quantities[7]. Within the de Broglie-Bohm theory all these quantities do have a different ontological status than position. Dürr *et al.* write (using spin as an example only):

> "Unlike position, spin is not *primitive*, i.e., no *actual* discrete degree of freedom, analogous to the *actual* positions of the particles, added to the state description in order to deal with "particles with spin". Roughly speaking, spin is *merely* in the wave function." (Dürr *et al.* 1996, p.11)

In common jargon these properties are called "contextual", i.e. the measurement does not reveal a pre-existing value of a system-property but depends crucially on the experimental arrangement (the "context").[8]
Thus, in general, "measurements" do not measure anything in the closer meaning of the term. The only exception being of course position measurements, and, in some sense momentum-measurements. The latter do indeed measure the asymptotic (Bohmian) velocities. Hence, the only properties of a "Bohmian particle" are its position and its velocity. Just as ψ is no classical field, the Bohmian particles are no classical particles, i.e. they are no bearers of properties other than position.
This reading of the observable concept throws new light on the Kochen-Specker "no-go" theorem, directed against hidden variable theories (see e.g. Mermin 1990). This theorem demonstrates, that a consistent assignment of possessed values to all observables for a quantum mechanical state is not possible. However, if you allow for contextuality – as the de Broglie-Bohm theory does – you do not *expect* such an assignment to exist at all.
According to Dürr *et al.* (2004) the "naive realism about operators", i.e. the identification of operators with properties and the common talk about "measuring" operators, is the source of most of the confusion in the interpretation of quantum mechanics. However, given what we have said above, it may appear puzzling why operators can play such a prominent role in the usual formulation of quantum mechanics and how exactly they relate to the Bohmian formulation. In Dürr *et al.* (2004) it is shown how operators naturally arise in the de Broglie-Bohm theory. They are derived quantities which are coding the probability distributions for certain "measurement-like" (p.11) experiments. This leads us to the next section which is devoted to a discussion of how the de Broglie-Bohm theory treats "measurements" and in particular how it solves the measurement problem.

[7] In fact, Holland (1993, p.91ff) introduces "local expectation values" for these quantities which are supposed to correspond to their "actual" value along the trajectories. Averaged over the quantum equilibrium distribution these local expectation values reproduce the quantum mechanical predictions. However, one might object that these "properties" are redundant since the position is already enough to reproduce all experimental predictions of quantum mechanics. Further more they are not conserved along the Bohmian trajectories.

[8] In Dürr *et al.* (2004, p.64ff) it is argued that the term "contextual property" is actually misleading because it suggests that e.g. spin is still a "property". But "properties which are merely contextual are no properties at all" (Dürr *et al.* 2004, p.67).

3.3 How the de Broglie-Bohm theory solves the measurement problem

Let us first briefly recall the measurement problem of quantum mechanics. It can be stated in several ways, e.g. Maudlin (1995, p.7) offers the following formulation:[9]

> The following three claims are mutually inconsistent:
>
> A The wave-function of a system is *complete*, i.e. the wave-function specifies (directly or indirectly) all of the physical properties of a system.
>
> B The wave-function always evolves in accord with a linear dynamical equation (e.g. the Schrödinger equation).
>
> C Measurements of, e.g. the spin of an electron always (or at least usually) have determinate outcomes [...]

The argument runs like this: Given a two-valued observable S with eigenvectors ψ_1 and ψ_2. Let Φ_0 denote its wavefunction in the "ready-state" and Φ_1 (Φ_2) the state of the apparatus if the measurement yields ψ_1 (ψ_2). Hence, $\hat{U}(\psi_i \otimes \Phi_0) = \psi_i \otimes \Phi_i$ ($i \in \{1,2\}$) holds, with $\hat{U}$ the time evolution of the combined system. A general state will be a superposition:

$$\psi = c_1\psi_1 + c_2\psi_2$$

Now, given B, the action of $\hat{U}$ on this state yields:

$$\hat{U}(\psi \otimes \Phi_0) = c_1\psi_1 \otimes \Phi_1 + c_2\psi_2 \otimes \Phi_2 \tag{4}$$

While individual measurements always result in *either* the state Φ_1 *or* Φ_2, this is a superposition of different pointer states. Thus, in contrast to our experience quantum mechanics does not leave the joint object-apparatus system in a definite state.[10] According to assumption A the wave-function should specify every physical fact about the measurement device. Maudlin argues that, since the two Φ_i enter symmetrically, it is not clear by what argument one could attempt to show that the final state 4 represents one but not the other indicator state. Thus, assuming A and B contradicts C. Any resolution of this problem has to deny at least one of the above assumptions.
To deny A needs some sort of "hidden" (or actually "additional") variables. The de Broglie-Bohm theory is a prominent example for this strategy and we explain

[9] In fact, Maudlin (1995) introduces three slightly different formulations of the measurement problem. We refer only to the first formulation (hence, Maudlin labels the following propositions 1.A, 1.B and 1.C).

[10] Our argument relied on simplifying assumption like an ideal measurement and pure states for both, object and apparatus. One might suspect that the problem is only generated by these unrealistic conditions. However, even in the completely general case employing density operators (i.e. mixed states), non-ideal measurements, interactions with the environment etc.pp. the conclusion remains essentially unaltered (see Bassi and Ghirardi (2000) and Grübl (2003)).

how this solves the measurement problem further below. Also does the statistical or ensemble interpretation of Ballentine (1970) questions A. It takes the quantum state to be the description of the statistical properties of an ensemble of identically prepared objects only.
To deny B leads to so-called "collapse theories" which abandon the strict linear time evolution of the system. For example Ghirardi, Rimini and Weber (1986) have developed such a non-linear model which describes this mechanism. Also does von Neumann's proposal of a collapse of the wavefunction fall into this category. However, von Neumann (like all other standard presentations of quantum mechanics) did not specify the physical conditions under which the linear evolution fails.
Finally one may question C and the many-world interpretation can be construed as a solution of the measurement problem along this line.

Effective collapse in the de Broglie-Bohm theory

Now we turn in more detail to the de Broglie-Bohm theory and its resolution of the measurement problem. It denies A, i.e. introduces the particle position as additional variables to arrive at a complete state description. However, what is needed are not just "additional" variables but variables which supply the necessary means to distinguish different measurement outcomes.[11]
Quantum mechanics describes how a superposition state evolves into a sum of macroscopic distinct (i.e. non-overlapping) states, i.e. $(\psi_1 \otimes \Phi_1) \cdot (\psi_2 \otimes \Phi_2) \approx 0$. It just fails to distinguish the branch which corresponds to the actual measurement outcome. Within the de Broglie-Bohm theory the different measurement outcomes correspond to different *configurations* (e.g. pointer positions). The positions provide a record of the measurement outcome, or more generally they "yield an image of the everyday classical world" (Bell 2001, p.41).
Suppose for example that the measurement yields outcome "1", i.e. the initial position of the Bohm particle was such that the deterministic evolution developed into a configuration that lies within the support of $\psi_1 \otimes \Phi_1$. The Bohm particles will be guided by this state because the non-overlapping $\psi_2 \otimes \Phi_2$-part is dynamically irrelevant. Thus the de Broglie-Bohm theory provides a so-called "effective collapse" of the wavefunction. Given the quantum equilibrium hypothesis the probability for this effective collapse obeys Born's rule.

4 Relativistic and quantum field theoretical generalizations

Presumably the most common objection[12] against the de Broglie-Bohm theory is based on its non-locality and its apparent conflict with relativity and quantum

[11] Maudlin (1995 p.11) notes that therefore "additional" variables which would be really "hidden" (i.e. unobservable) would not help at all.
[12] A comprehensive discussion of objections against the de Broglie-Bohm theory can be found in Passon and Struyve, 2006

field theory. However, several "Bohm-like" models for relativistic quantum mechanics and quantum field theory do exist. Here we give a non-technical account of some of these models. But before doing so, we need to say a few words on the actual meaning of "Bohm-like".

4.1 What is a "Bohm-like" theory?

At first sight "Bohm-like" seems to mean "having trajectories" or even "having deterministic trajectories". Obviously this requirement is intended to capture the spirit of the de Broglie-Bohm theory. The task of developing e.g. a Bohm-like quantum field theory is then to reconcile this concept with the predictions of QFT.

This may even be possible (see for example the Bell-type models below), however, on closer inspection this requirement seems to be too narrow nevertheless. One only needs to consider the history of physics, where many important features of a given theory did not carry over to its generalization. In particular does QFT provides examples for the departure from concepts which were accepted in non-relativistic quantum mechanics. Or to put it differently: one should expect (or at least not exclude from the outset) new concepts to enter a theory if it is extended to new areas.

Another more reasonable demand for a quantum field theoretical generalization of the de Broglie-Bohm theory is that it (i) reproduces the predictions of QFT and (ii) includes the non-relativistic formulation as a limiting case. The last requirement seems necessary to regard a model as a generalization. In Sec.4.4 we will come back to this important question.

However, the existing models for "Bohm-like" QFT concentrate on still another feature of the de Broglie-Bohm theory. They suggest, that the essence of the de Broglie-Bohm theory is its "clear ontology", i.e. that it attributes "being" to certain entities. In common jargon, the theory possesses "beables". This term was coined by Bell (1976) and is meant in contrast to "observable" i.e. emphasizes that any observation (i.e. measurement) deserves no special role in the formulation of a fundamental theory. In Bell's own words:

> "In particular we will exclude the notion of "observable" in favor of that of "beable". The beables of the theory are those elements which might correspond to elements of reality, to things which exist. [...] Indeed observation and observers must be made out of beables." (Bell 1986, p.174)

The beables of the non-relativistic de Broglie-Bohm theory happen to be particles (moving on continuous and deterministic trajectories). In what follows we will also come across field-beables and indeterministic dynamics in "Bohm-like" theories. As long as this beables provide the means to record measurement outcomes they can be used to build a Bohm-like model.

4.2 The Bohm-Dirac theory

We begin with the question of a relativistic generalization. Already in Bohm (1953) an extension of the de Broglie-Bohm theory to the Dirac equation was given. The strategy here is analogous to the non-relativistic case. Solutions of the Dirac equation fulfill a continuity equation with a time-like current. The spatial part of this current reads $\psi^\dagger \alpha_k \psi$. In addition the density $\rho = \psi^\dagger \psi$ (the appropriate *quantum equilibrium distribution*) is positive definite. Thus, similar to the non-relativistic case a particle velocity can be defined by the ratio of these two quantities:

$$\frac{dQ_k}{dt} = \frac{\psi^\dagger \alpha_k \psi}{\psi^\dagger \psi} \tag{5}$$

$$\text{with:} \qquad \alpha_k^i = 1 \otimes \cdots \otimes \alpha^i \otimes \cdots \otimes 1 \quad \text{and: } \alpha^i = \begin{pmatrix} 0 & \sigma_i \\ \sigma_i & 0 \end{pmatrix}$$

In this way the description is complemented by the configuration, i.e. the beables of this theory are particles as in the non-relativistic formulation.

However, in the many-particle case this theory is not Lorentz covariant since it uses a common time for all particles. The frame-of-reference in which $\rho = \psi^\dagger \psi$ holds is distinguished (Berndl *et al.* 1996). But this non-covariance is only relevant on the level of individual particles. The statistical predictions of the Bohm-Dirac theory are the same as for the usual Dirac theory because (i) by construction it is ensured that they hold in the distinguished frame and (ii) they transform properly under Lorentz transformations. Hence, the preferred frame-of-reference can not be identified experimentally.

In fact, as shown by Dürr *et al.* (1999), it is even possible to formally restore Lorentz invariance for the Bohm-Dirac theory by introducing additional structure. Dürr *et al.* introduce a preferred slicing of space-time, determined by a Lorentz invariant law.

In order to deal with anti-particles one might invoke the Dirac-sea concept, i.e. introduce particle beables for every negative energy state (Bohm and Hiley 1993, p.276).

Other approaches to develop a relativistic de Broglie-Bohm theory use the concept of the multi-time wavefunction $\psi(q_1, t_1, \cdots, q_N, t_N)$, i.e. introduce a different time variable for each particle. However, the resulting set of coupled Dirac equations can only be solved in the absence of interaction potentials. See Tumulka (2006) and the references therein for a more detailed discussion of these models.

However, it is generally agreed that the unification of quantum mechanics and relativity needs a quantum field theoretical framework anyway. We therefore turn to the field theoretical generalizations of the de Broglie-Bohm theory. Here several competing models do exist.

4.3 Quantum field theoretical generalizations

We have learned in Sec. 4.1, that the *beable* is the decisive quantity in a Bohm-like theory. Hence, the different models for a quantum field theoretical generalization of the de Broglie-Bohm theory can be classified according to the beables they employ. Roughly the models fall into the following three categories:

Field-beables for bosons and particle beables for fermions

Already in his seminal paper in 1952 Bohm presented a way of generalizing his causal interpretation to the electromagnetic field. The additional variables (or beables) were not particles but fields. The quantum state is thereby a wavefunctional which guides the field beable. This approach can be extended to the various bosonic fields (see e.g. Bohm 1984, Holland 1993, Kaloyerou 1996). For example the second-quantized real Klein-Gordon field is described by a wavefunctional $\Psi(\phi(\mathbf{x}),t)$, which satisfies the Schrödinger equation:

$$i\frac{\partial\Psi}{\partial t} = \int d^3x \left(-\frac{\delta^2}{\delta\phi^2} + (\nabla\phi)^2\right)\Psi. \tag{6}$$

The corresponding guidance equation for the field beable $\phi(\mathbf{x},t)$ reads

$$\frac{\partial\phi}{\partial t} = \frac{\delta S}{\delta\phi}, \tag{7}$$

where S is the phase of the wavefunctional Ψ.
In these models the configuration space is the infinite dimensional space of field configurations. Since there does not exist a Lebesgue volume measure on these spaces the rigorous definitions of an equivariant measure, i.e. the analogue of $|\psi(q)|^2 dq$, is problematic (Tumulka 2006, p.12).
For fermionic quantum fields Bohm *et al.* argue that a causal interpretation in terms of field beables cannot be constructed (Bohm, 1987) and (Bohm and Hiley 1993, p.276). Instead Bohm and Hiley propose to introduce particle beables for fermions according to the Bohm-Dirac theory mentioned above. In fact, models by Holland and Valentini which try to provide field-beables for fermions did not succeed (Struyve and Westman 2006, p.1).

Field-beables for bosons and no beable-status for fermions

Inspired by the difficulties to construct a Bohm-like theory for fermions with field-beables, Struyve and Westman (2006) propose a different direction. They note, that e.g. the property “spin” can be described in the de Broglie-Bohm theory without assigning a beable status to it. They suggest, that the same may be done for the fermionic degrees of freedom. Since fermions are always gauge-coupled to bosonic fields it is sufficient to introduce beables for the bosons.
Technically their work is similar to Bohm’s model with field-beables for bosons mentioned above. They introduce a specific representation for the bosonic field-operators and trace out the fermionic degrees of freedom. Their beables are the transversal part of the vector potential. In Struyve and Westman (2006) this approach is carried out for QED, but it has a natural extension to other gauge theories.

Struyve and Westman discuss in detail how this model accounts for an effective collapse, i.e. how the total wavefunctional evolves to a superposition of non-overlapping wavefunctionals. However, one might still worry if this model is capable to contain a record of the measurement outcome, for example in terms of pointer positions. They reply to this concern, that

> "(...) if we continue our quantum description of the experiment, the direction of the macroscopic needle will get correlated with the radiation that is scattered off (or thermally emitted from, etc.) the needle. Because these states of radiation will be macroscopically distinct they will be non-overlapping in the configuration space of fields and hence the outcome of the experiment will be recorded in the field beables of the radiation."(p.18)

We now turn to an approach which can be viewed as complementary to the Struyve-Westman model. While their model views fermions as an epiphenomenon, the Bell model we are going to discus next can be seen as tracing out the bosonic degrees of freedom (Struyve and Westman 2006, p.8).

Particle beables for fermions

John Bell (1986) presented a model for Hamiltonian quantum field theories with the fermion number as beable. He regarded this to be a natural generalization of the particle concept, since

> "The distribution of fermion number in the world certainly includes the positions of instruments, instrument pointers, ink on paper, ... and much much more." (p. 175)

Hence, to assign beable status to this quantity ensures a solution of the measurement problem.[13] This model is formulated on a spatial lattice with points enumerated by $l = 1, 2, \cdots, L$ (the time remains continuous). For each lattice site a fermion number operator is defined with eigenvalues $F(l) = 0, 1, 2, \cdots, 4N$ (N being the number of Dirac fields).
The "fermion number configuration" at each time is thus the list $n(t) = (F(1), \cdots, F(L))$.
While the non-relativistic de Broglie-Bohm theory regards (ψ, Q_i) to be the complete specification of the state of a system, this model considers the pair $(|\psi\rangle, n)$ (with $|\psi\rangle$ being the state vector).
The task is now to find the proper dynamics for this pair. For the state vector the usual evolution

$$\frac{d}{dt}|\psi(t)\rangle = \frac{1}{i}H|\psi(t)\rangle$$

is considered (in the following $\hbar$ is set to 1). Again this gives rise to a continuity equation:

$$\frac{d}{dt}P_n = \sum_m J_{nm} \tag{8}$$

[13] However, Bell acknowledges that this beable choice is everything but unique (p.179).

$$\text{with:} \qquad P_n = \sum_q |\langle n, q|\psi(t)\rangle|^2$$
$$J_{nm} = \sum_{q,p} 2\mathrm{Re}\langle\psi(t)|n, q\rangle\langle n, q| - iH|m, p\rangle\langle m, p|\psi(t)\rangle$$

Here q and p denote additional quantum numbers such that e.g. $|p, n\rangle$ forms a basis in Hilbert space. The n and m in the state specification denote the fermion number. Thus P_n is the probability distribution for the fermion number configuration n. While ordinary quantum mechanics (or quantum field theory) views this as the probability to *observe* the system in this state, Bell views it as the probability for the system to *be* in this state. Therefore it is his ambition to establish an analog to the guidance equation, i.e. to describe the time evolution of this *beable* irrespectively of its being observed or not.
Bell prescribes a stochastic evolution[14] for the fermion number with the jump rate T_{nm}, i.e. the probability to jump to the configuration n within the time span dt, given that the present configuration is m, is given by $T_{nm}dt$. Clearly the following equation holds:

$$\frac{dP_n}{dt} = \sum_m (T_{nm}P_m - T_{mn}P_n), \tag{9}$$

i.e. the change of P_n in time is given by the jumps $m \to n$ diminished by the jumps $n \to m$. However, Equ.9 must be reconciled with condition 8, i.e. the stochastic dynamics needs to obey the continuity constraint. This leads to the condition $J_{nm} = T_{nm}P_m - T_{mn}P_n$, which is for example satisfied by the choice:[15]

$$T_{nm} = \begin{cases} J_{nm}/P_m & \text{if} \quad J_{nm} > 0 \\ 0 & \text{if} \quad J_{nm} \leq 0 \end{cases}$$

Finally, the probability $T_{nn}dt$ for the system to remain in the same fermion number configuration is fixed by the normalization $\sum_m T_{mn}dt = 1$. Given an initial configuration of the fermion number in accordance with $P_n(t_0) = \sum_q |\langle n, q|\psi(t_0)\rangle|^2$ this model reproduces all predictions of ordinary quantum field theory.[16] The physical picture is that the world describes a random walk in the fermion-number configuration space; this random walk being biased by the state $|\psi(t)\rangle$.
Dürr *et al.* (2004b, 2005) developed a similar process in the continuum for more or less any regularized quantum field theory and call it “Bell-type quantum field theories”. While their model is continuous it still includes a random processes i.e. is non-deterministic. However, work of Colin (2003) suggests that it is also possible to construct a deterministic continuum limit. The difference between these two continuum versions of the Bell-model lies in the treatment of the vacuum. Dürr *et al.* take it to be the state with no particle-beables. In contrast does Colin’s

[14] Bell expected the indeterminism to disappear in the continuum limit.
[15] This choice is not unique, e.g. one may add solutions of the homogeneous equation.
[16] Bell notes that this includes also the outcome of the Michelson-Morley experiment, although this formulation relies on a particular division of space-time. Hence the violation of Lorentz invariance is not detectable (p.179).

model introduce particle beables for every negative energy solution, i.e. invokes the Dirac sea concept. Thereby the configuration space becomes infinite dimensional, i.e. does not possess a Lebesgue volume measure. As mentioned before in the context of field-beables this introduces problems for a rigorous definition of an *equivariant measure* (Tumulka 2006, p.15).

4.4 Some remarks on theory-generalization

In Sec.4.1 we have argued that having beables qualifies a theory as "Bohm-like". Further more we have used the expression "Bohm-like" and "generalization of the de Broglie-Bohm theory" synonymously. However, there seem to be reasonable distinctions between these two concepts. In the remainder of that paper we want to discuss the issue of theory generalization in some more detail. We will argue that being a "generalization of the de Broglie-Bohm theory" is actually a more restrictive property than being "Bohm-like" only. We investigate whether this may help to single out a candidate from the competing models discussed in the previous section. However, we will also see that this is complicated by the fact that the concept of "theory generalization" is more involved than usually considered.

Do all "Bohm-like" models generalize the de Broglie-Bohm theory?

So far we have been discussing "Bohm-like" QFT or actually "beable-QFT". However, we have already indicated in Sec. 4.1, that in order to regard these models as a "generalization" of the original theory it is reasonable to demand a specific relation between the non-relativistic formulation and these models. Very natural is the requirement that the Bohm-like QFT should include the non-relativistic de Broglie-Bohm theory as a *limiting case*. After all, there is no strict boundary between non-relativistic and relativistic physics and the corresponding theories should ideally merge to each other. We want to call this our preliminary criteria for "theory generalization".
Vink (1993, p.1811) investigates the relation between his generalized Bell-model and the original de Broglie-Bohm theory. He shows that the stochastic dynamics leads to the ordinary de Broglie-Bohm theory in the continuum limit. His argument is mathematically not rigorous but given that this model employs a particle-ontology from the outset it is certainly plausible to expect such a limit to exist.
The situation seems very different when it comes to field-beables; for example in the Struyve-Westman model. Given that there the fermionic degrees of freedom have no beable status it is not conceivable how to obtain the non-relativistic formulation as a limiting case. One may illustrate this with the example of the hydrogen atom. In the de Broglie-Bohm theory the physical picture of this system is a particle-beable (assigned to the electron) distributed according to $|\psi|^2$. In the Struyve-Westman model only the radiations degrees of freedom of the electromagnetic field have beable status and the "electron" is only an epiphenomenon. Therefore the Bohm-like QFT à la Struyve and Westman can not be viewed as

a generalization of the ordinary de Broglie-Bohm theory (in the above sense) but provides a complete reformulation of the non-relativistic theory.
Thus, the criteria whether a Bohm-like QFT includes the de Broglie-Bohm theory as a limiting case seems to allow an assessment of the different models. Rated by this measure the Bell-type models seem to be superior since they start with the same ontology as the non-relativistic formulation from the outset. But do we really have compelling arguments to make the non-relativistic formulation the touchstone for QFT generalizations? One could also be willing to modify the non-relativistic de Broglie-Bohm theory (e.g. along the lines sketched above in the hydrogen example). It seems reasonable to argue that not the non-relativistic formulation itself but only its predictions need to be recovered.
But there is even another twist in the above argument. Sofar we have employed a specific concept of "theory generalization" (the limiting case relation) and found that the field-beable approach has problems to cope with it. However, one may also ask how natural the requirement of the limiting case relation actually is. In fact these and related intertheory relations have been critically examined within the philosophy of science. We will therefore say a few words on this debate and its possible impact on our question.

What does it mean to "generalize" a theory?

Within the philosophy of science this question is part of the study of *intertheory relations* (Batterman 2005) and offers some surprises.
Traditionally this and related questions were framed in the context of "reductive relations" between theories, i.e. the question whether a given theory T_1 (the primary theory) reduces to T_2 (the secondary theory).[17] In some sense "theory generalization" is the inverse operation to "theory reduction". An early and influential treatment of theory reduction was given by Nagel (1961, Chapter 11) who viewed theory reduction essentially as a relation of deduction, i.e. the laws of the secondary theory should be derivable from the laws of the primary theory. However, this typically requires a translation of the descriptive terms of T_2 which are absent in T_1 into the T_1-language (so-called "bridge principles").
In reply to criticism against the highly idealized picture of the Nagelian account more sophisticated models of reduction have been developed (e.g. Schaffner (1967, 1969), Nickles (1973) and Hooker (1981)). Our above discussion used the notion, that a theory, T_1, reduces to an other, T_2, if T_2 is obtained as a *limiting case*, i.e. if there is a parameter, say ϵ, in the primary theory such that the laws of the secondary theory are obtained in the limit $\epsilon \to 0$. This is a modification of the Nagelian account due to Nickles (1973). The textbook example is the relation between special relativity and classical mechanics in the limit $(v/c)^2 \to 0$.
However, it has been shown that this notion of reduction can not account for many relevant cases. For example the mathematical physicists Sir Michael Berry noted with respect to this example, that

[17] Here we take "reduction" to be the move from the general (i.e. more fundamental) to the specific. In the philosophical literature it is often regarded the other way around.

> "(...) this simple state of affairs is an exceptional situation. Usually, limits of physical theories are not analytic: they are singular, and the emergent phenomena associated with reduction are contained in the singularity." (Berry 1994, p.599)

In such cases there is no smooth reduction relation between the corresponding theories, i.e. the secondary theory can neither be derived from the primary theory nor obtained as a limiting case, since the limit simply does not exist.[18] Examples investigated by Berry are the relation between wave and ray optics or quantum and classical mechanics.[19] In fact the classical limit of quantum mechanics belongs to the open foundational questions of the theory (see Landsman 2005 for an excellent overview).

Thus, there are many relevant cases in physics which intuitively count as "theory generalization" but fail to satisfy the limiting-case relation. If one is not willing to loose these cases one can not require this condition.

With respect to the relation between higher level and lower level (i.e. more fundamental) theories some authors argue for a relation called "emergence". The different versions of emergence roughly share the idea that "emergent entities (properties or substance) 'arise' out of more fundamental entities and yet are 'novel' or 'irreducible' with respect to them" (O'Connor and Wong (2002)). Another way to characterize emergence is simply by a denial of reduction (R-emergence) or a denial of supervenience[20] (S-emergence) (see Howard 2003, p.3ff).

However, if one denies the possibility to *reduce* a theory from a more fundamental level, the inverse move (i.e. the theory generalization) is affected as well. In what sense should a theory T_1 be regarded as a generalization of (i.e. being more "fundamental" than) a theory T_2 if it is not possible to recover T_2 from T_1? The whole talk about "higher level", "lower level" or being "more fundamental" becomes void and one seems to be left over with autonomous theories.

These brief remarks shall indicate that the concept of a "theory generalization" is more involved than usually considered. Thus, the failure of e.g. Bohm-like QFT with field-beables to recover the ordinary de Broglie-Bohm theory as a limiting case may be viewed rather as a generic feature in the relation between "higher" and "lower" level theories and not as a reason to reject this model.

[18] A simple example of a singular limit is given by Batterman (2005). The equation $x^2\epsilon + x - 9 = 0$ has two roots for any value of $\epsilon > 0$ but only one solution for the $\epsilon = 0$ case. Thus, the character of the behavior in the case $\epsilon = 0$ differs fundamentally from the character of its limiting (i.e. ϵ small but finite) behavior.

[19] Interestingly this is not taken as evidence against reduction per se. Berry states, that "what follows should not be misconstrued as antireductionist. On the contrary, I am firmly of the view [...] that all the sciences are compatible and that details links can be, and are being, forged between them. But of course the links are subtle [...]" (Berry 2001, p.4).

[20] Supervenience may be characterized as an ontic relation between structures, i.e. sets of entities together with properties and relations among them. A structure S_A is said to supervene on an other, say S_B, if the A-entities are composed of B-entities and the properties and relations of S_A are determined by properties and relations of S_B. It should be noted that neither does reduction entails supervenience nor the other way around.

It might still be possible to justify a certain beable choice based on the criteria that the relation between the corresponding Bohm-like QFT and the non-relativistic de Broglie-Bohm theory has desirable properties. However, this needs a more refined definition of "theory generalization". It seems very promising to investigate the Bohm-like quantum field theories as case studies for intertheory relations in order to learn more about both, "theory generalization" in general and the de Broglie-Bohm-program in particular.

5 Summary and conclusion

The non-relativistic de Broglie-Bohm theory is able to give an observer independent account of all quantum phenomena. It solves the infamous measurement problem, or, to be more precise, there is no such problem in the de Broglie-Bohm theory. It serves as a counter example to the common claim that no description of quantum phenomena can be given which employs particles moving on continuous trajectories. However, like most alternative interpretations it is not experimentally distinguishable from standard quantum mechanics.
When it comes to relativistic and quantum field theoretical generalizations one first needs to agree upon what one actually means by a "Bohm-like" theory. Seemingly a theory needs to have deterministic trajectories to count as "Bohm-like". However, most Bohmians would suggest that the decisive property of the de Broglie-Bohm theory is that it attributes a "beable-status" to certain properties. As long as these beables provide the means to record measurement outcomes they can be used to build a Bohm-like model. Particle beables are just a specific example for this strategy. For relativistic and quantum field theoretical generalizations several competing models do exist. These display a surprising flexibility with respect to the "beable-choice". Some models stick to a particle ontology while others introduce field-beables. Further more there is no need to introduce beables for all particle species and e.g. the Struyve-Westman model does without a beable status for fermions.[21]
A further investigation of the relation between these different models and the original de Broglie-Bohm theory seems to be an interesting case-study for what has been called "intertheory relations" in the philosophy of science. Possibly an assessment of these models could be based on the result.
Be that as it may, the common claim that the de Broglie-Bohm theory is incompatible with quantum field theory is certainly incorrect. Agreed, all these models have a "cooked-up" flavor, but this is due to the fact that their task is to reproduce the predictions of existing theories. These existing theories work FAPP (for all practical purposes) and the ambition of "Bohm-like" reformulations is not to extend their predictive power but to put them on a conceptually firm basis.
Now, does this mean that every physicist should be a Bohmian? Certainly not. But those who reject this possible quantum world should use correct arguments.

[21] The question whether all particles (should) have beable status is also addressed in Goldstein *et al.* (2005).

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References

Bacciagaluppi, G. and Valentini, A. (2006), *A tower of Babel: Quantum Theory at the 1927 Solvay Conference*, forthcoming.

Ballentine, L. E. (1970), *The Statistical Interpretation of Quantum Mechanics*, Rev. Mod. Phys. **42**, 358.

Bassi, A. and Ghirardi, GC. (2000), *A General Argument Against the Universal Validity of the Superposition Principle*, Phys. Lett. **A 275** 373 and quant-ph/0009020.

Batterman, R. (2005), *Intertheory Relations in Physics*, The Stanford Encyclopedia of Philosophy (Winter 2005 Edition), Edward N. Zalta (ed.), http://plato.stanford.edu/archives/win2005/entries/physics-interrelate/

Bell, J. S. (1976), *The Theory of Local Beables*, Epistem. Lett. **9**, 11.

Bell, J. S. (1986), *Beables for quantum field theory*, Phys. Rep. **137**, 49-54, page numbering according to the reprint in Bell (2004), 173-180.

Bell, J. S. (2001), John S. Bell on the Foundations of Quantum Mechanics, M. Bell, K. Gottfried and M. Veltman (eds), World Scientific: Singapore.

Bell, J. S. (2004), *Speakable and Unspeakable in Quantum Mechanics*, 2nd edition Cambridge University Press.

Berndl, K., D. Dürr, S. Goldstein, G. Peruzzi and N. Zanghi (1995), *On the Global Existence of Bohmian Mechanics*, Commun. Math. Phys. **173** 647-674 and quant-ph/9503013.

Berndl, K., D. Dürr, S. Goldstein and N. Zanghi (1995), *EPR-Bell Nonlocality, Lorentz Invariance, and Bohmian Quantum Theory*, Phys. Rev. **A 53** 2062-2073 and quant-ph/9510027.

Berry, M. (1994), *Asymptotics, Singularities and the reduction of Theories*, Proc. 9th Int. Cong. Logic, Method. and Phil. of Sci. IX, 597-607, eds. D Prawitz, B Skyrms and D Westerstahl.

Berry, M. (2001), *Chaos and the semiclassical limit of Quantum mechanics (is the moon there when somebody looks?)*, in Quantum Mechanics: Scientific perspectives on divine action (eds: Robert John Russell, Philip Clayton, Kirk Wegter-McNelly and John Polkinghorne), Vatican Observatory CTNS publications, pp 41-54.

Bohm, D. (1952), *A suggested interpretation of the quantum theory in terms of "hidden" variables*, Phys. Rev. **85**, 166 (part I) and 180 (part II).

Bohm, D. (1953), Prog. Theo. Phys. **9**, 273-87.

Bohm, D. and Vigier, J.-P. (1954), *Model of the Causal Interpretation of Quantum Theory in Terms of a Fluid with Irregular Fluctuations*, Phys. Rev. **96** 208.

Bohm, D. and Hiley, B. J. (1984), *Measurement as understood through the quantum potential approach*, Found. Phys. **14**, 255.

Bohm, D., Hiley, B. J. and Kaloyerou, P. N. (1987), *An ontological basis for the quantum theory*, Phys. Rep. **144**, 349.

Bohm, D. and Hiley, B. J. (1993), *The Undivided Universe*, Routledge, New York.

Cabello, A. (2000), *Bibliographic guide to the foundation of quantum mechanics and quantum information*, quant-ph0012089 (12th version, November 2004).

Colin, S. (2003), *A deterministic Bell model*, Phys. Lett. A 317, 349 and quant-ph/0310055.

Cushing, J.T. and McMullins E. (eds.) (1987), *Philosophical Consequences of Quantum Theory*, University of Notre Dame Press, Indiana.

Cushing, J.T. (1994), Quantum mechanics: Historical contingency and the Copenhagen hegemony, The University of Chicago Press, Chicago.

de Broglie, L. (1927), *La structure atomique de la matière et du rayonnement et la méchanique ondulatoire*, reprint in *La Physique Quantique restera-t-elle Indetérministe?*, Gauthier Villars, Paris 1953.

Deotto, E. and Ghirardi, GC. (1998), *Bohmian Mechanics Revisited*, Found. of Phys., Vol. **28**, No.1 1-30.

Dürr, D., Goldstein, S. and Zanghi, N. (1992), *Quantum equilibrium and the Origin of Absolute Uncertainty*, Journal of Statistical Physics, **67** 843.

Dürr, D., Goldstein, S. and Zanghí, N. (1996), *Bohmian Mechanics and the Meaning of the Wave Function*, in *Experimental Metaphysics - Quantum Mechanical Studies in Honor of Abner Shimony*, R. S. Cohen, M. Horne and J. Stachel (Eds.), Boston Studies in the Philosophy of Science, Kluwer. The page numbering refers to the quant-ph/9512031 version.

Dürr, D., Goldstein, Münch-Berndl, K., S. and Zanghí, N. (1999), *Hypersurface Bohm-Dirac models*, Phys. Rev. A **60**, 2729-2736 and quant-ph/9801070.

Dürr, D. (2001), *Bohmsche Mechanik als Grundlage der Quantenmechanik*, Springer, Berlin.

Dürr, D. , Goldstein, S. and Zanghi, N. (2004), *Quantum Equilibrium and the Role of Operators as Observables in Quantum Theory*, Journal of Statistical Physics 116, 959-1055. The page numbering refers to the preprint version quant-ph/0308038.

Dürr, D., Goldstein, S., Tumulka, R. and Zanghi, N. (2004b), *Bohmian Mechanics and Quantum Field Theory*, Phys. Rev. Lett. **93** 090402 and quant-ph/0303156.

Dürr, D., Goldstein, S., Tumulka, R. and Zanghi, N. (2005), *Bell-Type Quantum Field Theories*, J.Phys. A38 R1 and quant-ph/0407116.

Ghirardi, G. C., Rimini, A. and Weber, T. (1986), *Unified dynamics for microscopic and macroscopic system*, Phys. Rev. D **34**, 2, 470.

Goldstein, S. (1996), *Bohmian Mechanics and the Quantum Revolution*, Synthese **107**, 145-165 and quant-ph/9512027.

Goldstein, S. (2002), *Bohmian Mechanics*, The Stanford Encyclopedia of Philosophy (Winter 2002 Edition), Edward N. Zalta (ed.), http://plato.stanford.edu/archives/win2002/entries/qm-bohm/

Goldstein, S., Taylor, J., Tumulka, R. and Zanghi, N. (2005), *Are all particles real?*, Stud. Hist. Phil. Mod. Phys. **36** 103-112.

Grübl, G. (2003) *The quantum measurement problem enhanced*, Phys. Lett. A 316 153-158 and quant-ph/0202101.

Hooker, C. A. (1981), *Towards a General Theory of Reduction*, Dialogue **20**, 38-60, 201-236, 496-629.

Holland, P. R. (1993), *The Quantum Theory of Motion*, Cambridge University Press.

Howard, D. (2003), *Reduction and Emergence in the Physical Sciences: Some Lessons from Particle Physics-Condensed Matter Physics Debate*, manuscript: http://www.nd.edu/~dhoward1/Papers.html

Kaloyerou, P. N. (1996), *Ontological interpretation of boson fields* in *Bohmian Mechanics and Quantum Theory: An Appraisal*, J. T. Cushing, A. Fine and S. Goldstein (Ed.), Kluwer Academic Publishers, Dordrecht.

Landsman, N. P. (2005), *Between classical and quantum*, in Handbook of the Philosophy of Science, J. Earman und J. Butterfield (eds.), Elsevier *forthcoming* and quant-ph/0506082.

Nagel, E. (1961), *The Structure of Science*, London: Routledge.

Nickles, Th. (1973), *Two Concepts of Intertheoretic Reduction*, J. of Phil. 70, p. 181-201.

Maudlin, T (1995), *Three measurement problems*, Topoi **14**, 7-15.

Mermin, N. D (1990), *Simple Unified Form for the Major No-Hidden Variables Theorems*, Phys. Rev. Lett. **65**, 3373.

Myrvold, W. C. (2003), *On Some Early Objections to Bohm's Theory*, International Studies in the Philosophy of Science **17**, 7-24.

O'Connor, T. and Wong, H. Y. (2002), *Emergent Properties*, The Stanford Encyclopedia of Philosophy (Summer 2005 Edition), Edward N. Zalta (ed.), http://plato.stanford.edu/archives/sum2005/entries/properties-emergent/

Passon, O. (2004), *Bohmsche Mechanik*, Verlag Harri Deutsch: Frankfurt.

Passon, O. and Struyve, W. (2006), *Why isn't every physicist a Bohmian?*, in preparation (preliminary version under quant-ph/0412119).

Schaffner, K. (1967), *Approaches to Reduction*, Philosophy of Science 34: 137-147.

Schaffner, K. (1969), *The Watson-Crick Model and reductionism*, British Jour. for the Phil. of Science, **20**, 325-348.

Struyve, W and Westman, H. (2006), *A new pilot-wave model for quantum field theory*, *AIP Conference Proceedings* **844**, 321 and quant-ph/0602229.

Teufel, S. and Tumulka, R. (2005), *Simple Proof for Global Existence of Bohmian Trajectories*, Commun. Math. Phys. 258 349-365 and math-ph/0406030.

Tumulka, R. (2006), *The "Unromantic Pictures" of Quantum Theory*, quant-ph/0607124.

Valentini, A. (1991), *Signal-locality, uncertainty, and the subquantum H-theorem* , part I: Phys. Lett. A **156**, No.1-2, 5. part II: Phys. Lett. A **158**, No.1-2, 1.

Valentini, A. (1992), *On the Pilot-Wave Theory of Classical, Quantum and Subquantum Physics*, PhD. Thesis, International School for Advanced Studies, Trieste.

Vink, J. C. (1993), *Quantum mechanics in terms of discrete beables*, Phys. Rev. A **48** 3 1808.